Estimating Partisan Advantage

1. Context and Focus

*[This section will:*

* *enumerate a broad cross section of factors one might consider when evaluating how good or bad a redistricting plan is identifying partisan/political considerations as just one of many*
* *say that I’m only focused on partisan/political considerations*
* *then explain that even w/in that area there are several things to consider, including measures of gerrymandering/packing (declination, lopsided outcomes, and mean–median), bias, and responsiveness*
* *say again that I’m only focused on the “bias” subcategory – what I labelled the local and non-local in v1*
* *but because “bias” and “partisan bias” can have precise meanings to some people in some contexts, I will use the term “partisan advantage” instead.*

*so we can start Section 2 narrowly focused on that.]*

2. [Framework]

*[This draft section consolidates the concepts in previous Sections 1 (Definition of Partisan Bias), Appendix A (Interpreting the Efficiency Gap & Gamma), and Appendix B (Classifying Seats–Votes Curves) as a framework for analyzing the 2012, 2020, and hypothetical plans in Sections 3–5.*

*This is still incomplete and rough. I’m just trying to sketch out the main ideas. Ellipses (…) are where I will probably add/add back content.]*

## 2.1. Definition of Partisan Advantage

…

*Partisan advantage is the difference between the ideal and actual seat shares.*

…

To formalize this, call the ideal seats–votes relationship , where and the resulting are the two-party Democratic vote share and seat share, respectively.[[1]](#footnote-1) As you will see below, there are many candidates for this expected seats-votes function.[[2]](#footnote-2)

Similarly, call the actual seats–votes resultsand , the statewide vote share[[3]](#footnote-3) and the resulting seat share, respectively.

In practical terms, partisan advantage is simply the difference between these two:

(1)

In other words, partisan advantage is the difference between the share of seats that *should* be won and the share of seats *actually* won. Note: One can use a composite of prior elections – and the resulting statewide vote share and the district-by-district vote shares – as a proxy for a not-yet-held election to infer the *likely* seats–votes curve and use that to estimate and .[[4]](#footnote-4)

…

## 2.2. Properties of Measures

We wish to find measures of partisan advantage that respect the following properties:

* Property 1: The unit of measure is a difference of seat shares.[[5]](#footnote-5)  
    
  Metrics that don’t compare the difference between actual seat shares and some ideal measure some other aspect of a plan, but they don’t measure partisan advantage directly.
* Property 2: Super-proportional outcomes can’t favor the minority party.

1. If and , a valid measure of partisan advantage will not indicate that the plan favors Republicans.[[6]](#footnote-6)
2. Similarly, if and , a valid measure of partisan advantage will not indicate that the plan favors Democrats.

In graphical terms, what this means in Fig. 1 is that a valid measure of partisan advantage can’t classify points D1 or D2 as favoring Republicans or points R1 or R2 as favoring Democrats.

Chart, scatter chart

Description automatically generated

Fig. 1. S(V) space with 2-proportionality ideal (dashed line)

Plans where a party gets more than half the seats when they receive more than half the votes may or may not be considered biased in favor of that party *depending on which ideal function you choose.* More on that below.

* Property 3: Sub-proportional outcomes favor the minority party.[[7]](#footnote-7)

1. If and , a valid measure of partisan advantage will indicate that the plan favors Republicans.
2. Similarly, if and , a valid measure of partisan advantage will indicate that the plan is biased in favor of Democrats.

In graphical terms, what this means is that that a valid measure of partisan advantage must classify points D3 and D4 as favoring Republicans and points R3 and R4 as favoring Democrats.

D4 and R4 are anti-majoritarian plans where one party gets more than half the votes but less than half the seats—a subset of sub-proportional plans.  
  
Another way to express this property is that for any ideal seats­–votes curve, , it must be the case that for any , , and for any .[[8]](#footnote-8) Together the ideal and the line of proportionality create the zone of acceptability noted in Property 5.

* Property 4: Super-ideal outcomes favor the majority party.

1. If is the ideal seats–votes relationship, , and , then a valid measure of partisan advantage will classify the plan as favoring Democrats.
2. Conversely, if and , then a valid measure of partisan advantage will classify the plan as favoring Republicans.

In graphical terms, this means that a valid measure of partisan advantage must classify points D1 and R1 as favoring Democrats and Republicans, respectively.

* Property 5: Outcomes between proportional and ideal inclusive have acceptable bias not unduly favoring either party.

1. If , and , then a valid measure of partisan advantage will classify the plan as not unduly favoring either party.
2. Similarly, if and , then a valid measure of partisan advantage will classify the plan as not unduly favoring either party.

In graphical terms, this means that a valid measure of partisan advantage must classify points D2 and R2 as favoring as not unduly favoring either party.

* Property 6: Locality

While a seats–votes curve shows the likely seat share over all theoretically possible vote shares,[[9]](#footnote-9) only a small range around the typical statewide vote share are likely in practice.[[10]](#footnote-10) Hence, valid measures of partisan advantage measure it around the typical statewide vote share without recourse to aspects of the seats–votes curve outside that range.

This last property is analogous to the *principle of locality* in physics.[[11]](#footnote-11) Absent some theory and some empirical evidence to support it, there is no reason to believe that any metric that measures something far away from the likely statewide vote share measures anything related to partisan advantage close to it.

## 2.3 Candidate Measures

*[This section is very much a work in progress, but I wanted to flesh out your ideas about ideal S(V) functions.]*

With one modification discussed later, these three metrics are valid measures of partisan advantage as we have defined it:

* Proportional ()
* Efficiency gap (), and
* Gamma ()

They share the same underlying functional form:

(2)

where is an actual or idealized value of responsiveness:

* For proportionality, making the proportionality line where . On purely little ‘d’ democratic principles one might say that this the ideal seats–votes relationship.
* In contrast, the efficiency gap embeds an ideal responsiveness of 2—a two times winner’s bonus (). One can argue that this comports better with how single-member districts actually perform in practice.[[12]](#footnote-12)
* The gamma measure uses the responsiveness measured at the statewide vote share.

Moreover, since these measures are all evaluated at the statewide vote share and do not depend on counterfactuals outside the local range, we expect the signals from these metrics to match *a priori* expectations of partisan advantage even in states that are unbalanced politically.

Other candidates (from your email, lightly edited):

* Any seats-votes curve between 1-prop and 2-prop will work. More generally, you could take S(v)=kv and declare that any seat curve between 1-prop and k-prop is fine.
* Or you can take a non-linear S(v), say S(v)=1 for any v>0.5 (winner take all), and declare that actually, anything that is at least proportional is acceptable.
* Or your S(v) can be funky too: Say S(v)=1 for v in (0.5 to 0.6), then S(v) decreasing linearly with slope 1 down to S(0.8)=0.8, thereafter S(v)=v for v>0.8… and then any rule with s(v) in between v and S(v) also satisfies your axioms.

More non-linear possibilities (from Grofman and Tufte):

* One might specify the ideal seats–votes relationship as the “cube law” where:  
    
   (3)

which reduces to:

(4)

* This can, again, be generalized to:  
    
   (5)

I don’t know if or how that can be reduced to the functional form.

All of these are possible ways to express the ideal translation of votes into seats. We will examine three below – proportional (), efficiency gap (), and gamma () – in addition to four other prominent measures of bias – seats bias (), votes bias (), geometric seats bias (), and global symmetry () – and the three measures of symmetry discussed in the previous section – declination (), lopsided outcomes (), and mean–median ().

# 3–5. Analysis of 2012, 2020, and Hypothetical Plans

*[Largely as before but edited to be consistent with new Sections 1–2.]*

References

*[Expanded but still partial & probably not properly formatted]*

Dave’s Redistricting App (<https://bit.ly/3FI3AHa>).

Michael D. McDonald, Daniel B. Magleby, Jonathan Krasno, Shawn J. Donahue, and Robin Best, “Making a Case for Two Paths Forward in Light of *Gill v. Whitford*,” Election Law Journal: Rules, Politics, and Policy, Dec 2018, 315-327, [http://doi.org/10.1089/elj.2018.0527](https://doi.org/10.1089/elj.2018.0527)

Bernard Grofman, “Measures of Bias and Proportionality in Seats-Votes Relationships,” Political Methodology 9, no. 3 (1983): 295–327, <http://www.jstor.org/stable/25791195>.

Anthony McGann, Charles Anthony Smith, Michael Latner, and Alex Keena, (2016). *Gerrymandering in America: The House of Representatives, the Supreme Court and the Future of Popular Sovereignty*, Cambridge University Press, <https://doi.org/10.1017/CBO9781316534342>

John F. Nagle and Alec Ramsay, “On Measuring Two-Party Partisan Bias in Unbalanced States,” Election Law Journal: Rules, Politics, and Policy, Mar 2021, 116-138, <http://doi.org/10.1089/elj.2020.0674>

Nicholas Stephanopoulos and Eric McGhee, “The Measure of a Metric: The Debate Over Quantifying Partisan Gerrymandering,” 70 Stanford Law Review 1503 (2018).

Edward R. Tufte, “The Relationship between Seats and Votes in Two-Party Systems,” The American Political Science Review 67, no. 2 (1973): 540–54, <https://doi.org/10.2307/1958782>.

Samuel Wang, Sandra J. Chen, Richard Ober, Bernard Grofman, Kyles Barnes, and Jonathan Cervas, “Turning Communities Of Interest Into A Rigorous Standard For Fair Districting” (March 14, 2021). Stanford Journal of Civil Rights and Civil Liberties, Forthcoming, Available at SSRN: <https://ssrn.com/abstract=3828800>.

Gregory S. Warrington, “A Comparison of Partisan-Gerrymandering Measures,”Election Law Journal: Rules, Politics, and Policy, Sep 2019, 262-281, <http://doi.org/10.1089/elj.2018.0508>.

[end]

1. We use two-party Democratic vote shares by convention. Two-party Replication vote shares are simply . [↑](#footnote-ref-1)
2. Not just proportionality! [↑](#footnote-ref-2)
3. John Nagle has called this <V> in the past. [↑](#footnote-ref-3)
4. We use John Nagle’s base methodology of fractional seat probabilities described in <https://lipid.phys.cmu.edu/nagle/Technical/FractionalSeats2.pdf> and a seats–votes curve inferred using proportional shift described in <https://lipid.phys.cmu.edu/nagle/Technical/2019-04-19%20-%20Measuring%20Redistricting%20Bias%20&%20Responsiveness.pdf>. [↑](#footnote-ref-4)
5. In my first draft, I focused on the input parameters to candidate functions. In this draft, I’m focused on the return type. [↑](#footnote-ref-5)
6. To make formulas easier to write, we represent percentages as [0–1] fractions in the body text. Except as noted, in tables and figures, we show them as percentages. [↑](#footnote-ref-6)
7. Someone wanting to protect minority rights might argue that sub-proportional but not anti-majoritarian outcomes don’t unduly (dis)favor either party. Our stick in the ground is that proportionality is a minimum bar for the majority party. [↑](#footnote-ref-7)
8. The and versus simple and allow the ideal to be proportionality (). [↑](#footnote-ref-8)
9. Because statewide vote shares tend to not fall much outside the range [0.4, 0.6], we only infer the points of the seats–votes curve for the range [0.25, 0.75]. [↑](#footnote-ref-9)
10. As will be described below, we use a 5% range that brackets the statewide vote share, because the average uncertainty for the seats–votes curves in (Nagle and Ramsay, 2021) was roughly 2%. [↑](#footnote-ref-10)
11. <https://en.wikipedia.org/wiki/Principle_of_locality> [↑](#footnote-ref-11)
12. (Stephanopoulos & McGhee, 2018): “a fourth parameter is *empirical correspondence*. That is, the electoral ideal implied by a metric should not be too different from the American historical norm. Otherwise the measure would imply that most American plans have been gerrymanders—and its adoption would be so disruptive as to be infeasible.” [↑](#footnote-ref-12)